INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 2.13-2.14]

POLYNOMIAL MODELS

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. [Maximum mark: 9]

Consider the arithmetic sequence 13, 18, 23, 28, 33, 38, …

The terms of the sequence lie on the line L.

(a) Find

(i) the n-th term u_n of the sequence in terms of n

(ii) the equation of the line L in the form $y = mx + c$. [3]

(b) Write down a comment about the gradient m of the line L in the context of the arithmetic sequence. [1]

(c) Determine whether the points A(-1, 3), B(10, 58), C(0.2, 9) lie on the line. [3]

(d) Write down a comment about the y-coordinates of A, B, C in the context of the arithmetic sequence. [2]

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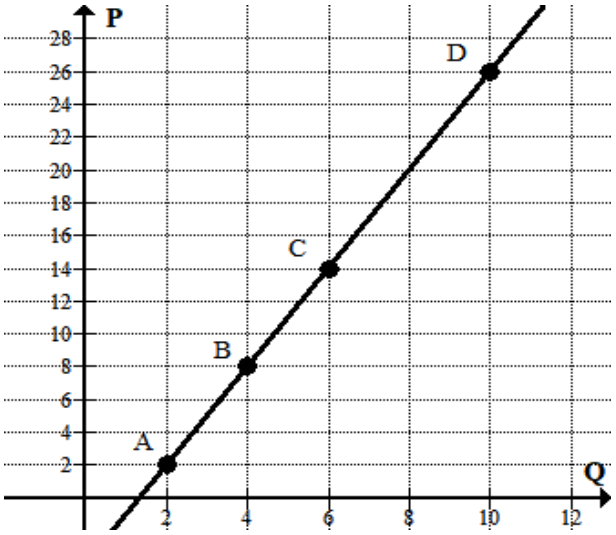
2. [Maximum mark: 8]

Consider the point series

Q	2	4	6	10
P	2	8	14	26

The following diagram shows that the four points of the series line on a line L.



Select the points A(2,2) and D(10,26) to find the linear equation of P in terms of Q, by using two different methods.

(a) Find the gradient of the line segment [AD} and hence find the equation of L. [3]

(b) Given that the points A and D lie on the line $P = aQ + b$, write down two linear equations in a and b and **hence** find the equation of L. [3]

(c) Verify the two remaining points lie on L. [2]

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3. [Maximum mark: 8]

Express each of the following quadratics in the form $y = ax^2 + bx + c$ based on the information given.

(a) $y = 2x^2 + 3x + c$ with y-intercept (0, 7). [1]

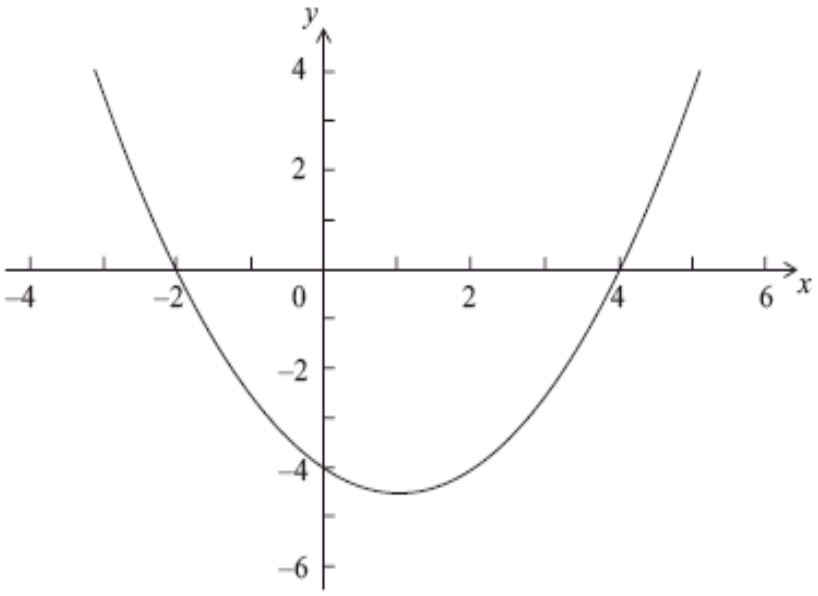
(b) $y = 2(x - p)(x - q)$ with x-intercepts (1, 0) and (5, 0). [2]

(c) $y = 2(x - h)^2 + k$ with vertex (3, 5). [2]

(d) $y = a(x - 3)^2 + 2$ with y-intercept (0, 20). [3]

4. [Maximum mark: 8]

Let $f(x) = p(x-q)(x-r)$. Part of the graph of f is shown below.



The graph passes through the points (–2, 0), (0, –4) and (4, 0).

(a) Write down the value of q and of r. [2]

(b) Write down the equation of the axis of symmetry. [1]

(c) Find the value of p. [3]

(d) Express the function in the form $f(x) = ax^2 + bx + c$. [2]

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5. [Maximum mark: 8]

Consider two different quadratic functions of the form $f(x) = 4x^2 - qx + 25$. The graph of each function has its vertex on the x-axis.

(a) Find both values of q. [3]

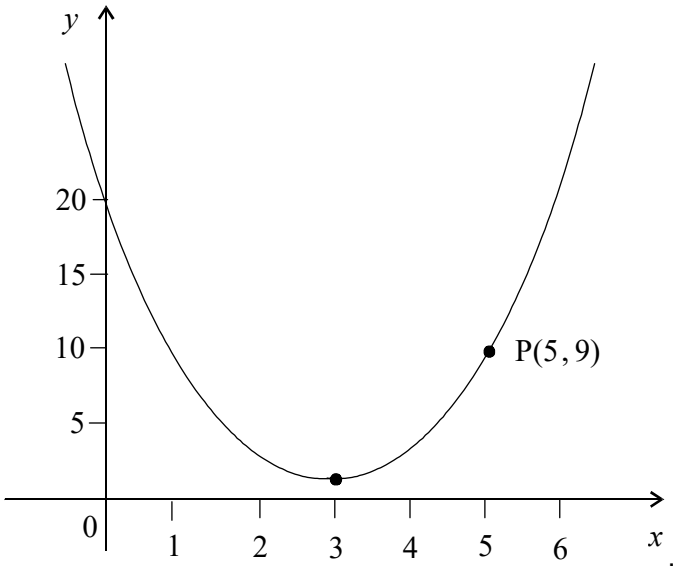
(b) For the greater value of q, solve $f(x) = 0$. [1]

(c) Find the coordinates of the point of intersection of the two graphs. [2]

(d) Sketch the two graphs on the same diagram. [2]

6. [Maximum mark: 7]

The diagram shows part of the graph of the curve $y = a(x - h)^2 + k$, where $a, h, k \in \mathbb{Z}$.



(a) The vertex is at the point $(3, 1)$. Write down the value of h and of k. [2]

(b) Write down the equation of the axis of symmetry. [1]

(b) The point $P(5, 9)$ is on the graph. Show that $a = 2$. [3]

(c) Hence show that the equation of the curve can be written as $y = 2x^2 - 12x + 19$. [1]

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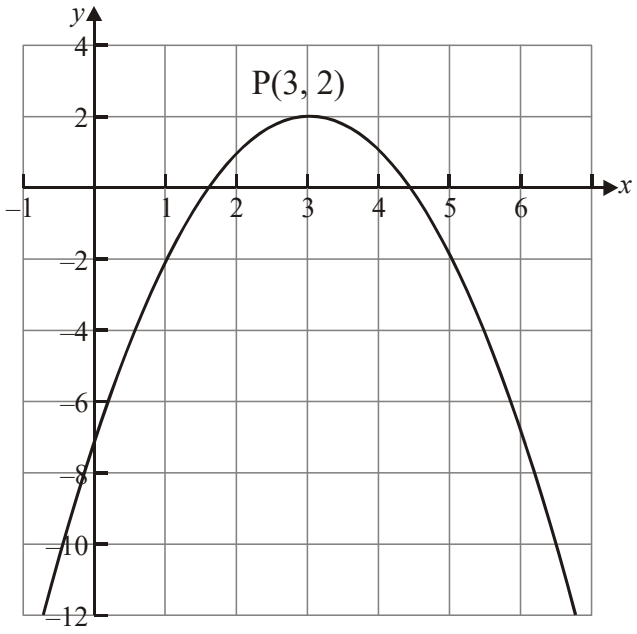
7. [Maximum mark: 6]

The function $f(x)$ is defined as $f(x) = -(x-h)^2 + k$. The diagram below shows part of the graph of $f(x)$. The maximum point on the curve is $\mathrm{P}(3, 2)$.



(a) Write down the value of (i) h (ii) k. [2]

(b) Write down the possible values of $f(x)$. [2]

(c) Given that $f(2) = a$, write down the other value of x such that $f(x) = a$. [1]

(d) Show that $f(x)$ can be written as $f(x) = -x^2 + 6x - 7$. [1]

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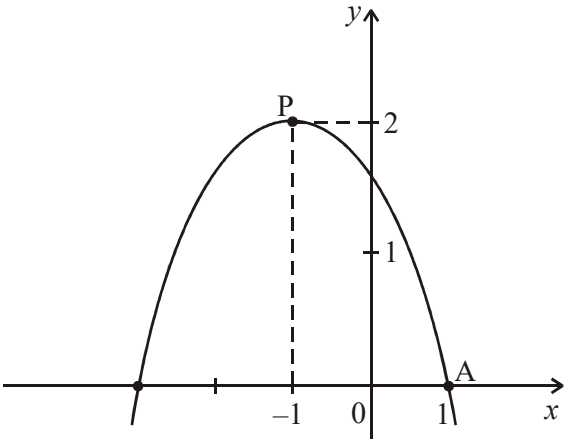
8. [Maximum mark: 8]

The diagram shows part of the graph of $y = a\,(x - h)^2 + k$. The graph has its vertex at P, and passes through the point A with coordinates $(1, 0)$.



(a) Write down the value of (i) h (ii) k [2]

(b) Calculate the value of a. [3]

(c) Express the quadratic function in the forms

(i) $y = a\,(x - p)\,(x - q)$

(ii) $y = ax^2 + bx + c$. [3]

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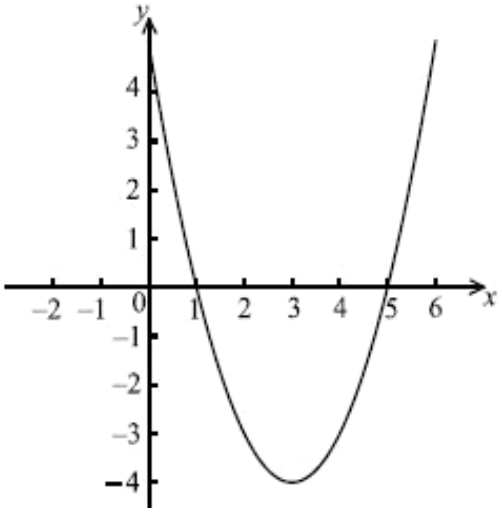
9. [Maximum mark: 8]

The following diagram shows part of the graph of a quadratic function, with equation in the form $f(x) = (x - p)(x - q)$, where $p, q \in \mathbb{Z}$.



(a) Write down

(i) the value of p and of q

(ii) the equation of the axis of symmetry of the curve. [2]

(b) Find the equation of the function in the form $f(x) = (x - h)^2 + k$, where $h, k \in \mathbb{Z}$. [2]

(c) Express the quadratic function in the form $f(x) = ax^2 + bx + c$, where $a, b, c \in \mathbb{Z}$. [2]

(d) Solve the inequality $f(x) < 0$. [2]

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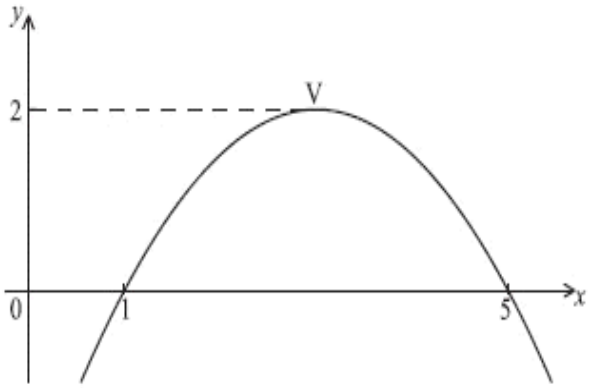
10. [Maximum mark: 7]

Part of the graph of the function $f(x) = d(x-m)^2 + p$ is given in the diagram below.

The x-intercepts are $(1, 0)$ and $(5, 0)$. The vertex is $V(m, 2)$.



(a) Write down the value of (i) m (ii) p [2]

(b) Find d. [2]

(c) Express the quadratic function in the form $f(x) = ax^2 + bx + c$. [2]

(d) Factorize $f(x)$. [1]

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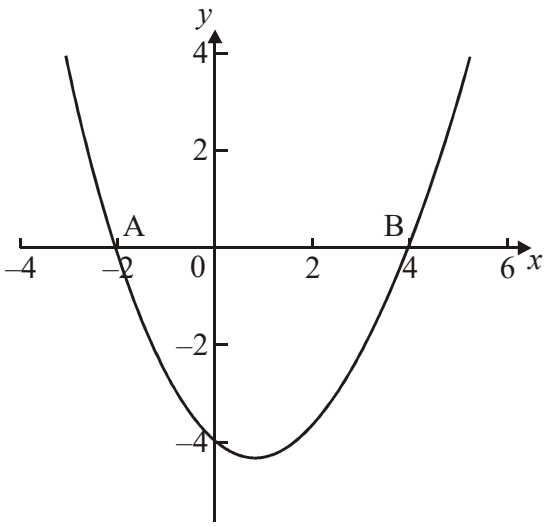
11. [Maximum mark: 8]

The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve intersects the x-axis at $A(-2, 0)$ and $B(4, 0)$. The curve of $y = f(x)$ is shown in the diagram below.



(a) Write down the value of p and of q. [2]

(b) Given that the point $(6, 8)$ is on the curve, find the value of a. [2]

(c) Write the equation of the curve in the form $y = ax^2 + bx + c$. [1]

(d) (i) Solve the inequality $y > 0$.

(ii) Given that x is a positive integer, write down the smallest value of x for which $y > 0$. [3]

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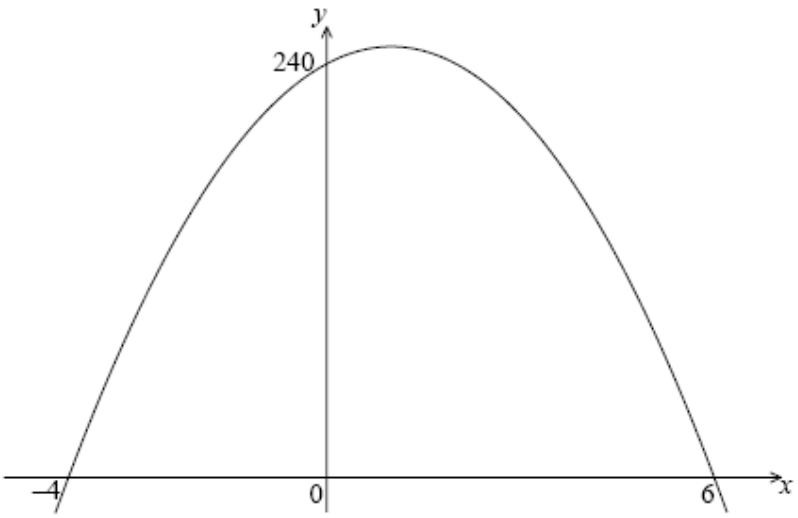
12. [Maximum mark: 8]

The following diagram shows part of the graph of a quadratic function f.



The x-intercepts are at $(-4, 0)$ and $(6, 0)$ and the y-intercept is at $(0, 240)$.

(a) Write down $f(x)$ in the form $f(x) = a(x - p)(x - q)$. [3]

(b) Find another expression for $f(x)$ in the form $f(x) = a(x - h)^2 + k$. [4]

(c) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. [1]

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13. [Maximum mark: 8]

Let $f(x) = ax^2 + bx + c$ where a, b and c are rational numbers.

(a) The point $P(-4, 3)$ lies on the curve of f. Show that $16a - 4b + c = 3$. [2]

(b) The points $Q(6, 3)$ and $R(-2, -1)$ also lie on the curve of f.

Write down two other linear equations in a, b and c. [2]

(c) **Hence** find $f(x)$. [2]

(d) Write $f(x)$ in the form $f(x) = a(x - h)^2 + k$, where a, h and k are rational numbers. [2]

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14. [Maximum mark: 5]

The following point series lies on a quadratic model $P = aQ^2 + bQ + c$

Q	**1**	**2**	**3**
P	**4**	**7**	**14**

(a) Write down three linear equations on a, b, c. [3]

(b) Hence find the quadratic model. [2]

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15. [Maximum mark: 7]

The following point series lies on a quadratic model $P = aQ^2 + bQ + c$

Q	**–2**	**1**	**2**
P	**0**	**0**	**12**

Find the quadratic model by using two different methods.

(a) By writing down three linear equations on a, b, c and solving the system. [4]

(b) By using factorization and the fact that two of the points are Q-intercepts. [3]

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16. [Maximum mark: 10]

The following point series lies on a quadratic model $P = aQ^2 + bQ + c$

Q	**0**	**3**	**6**
P	**1**	**10**	**1**

(a) By sketching a graph determine

(i) the vertex of the quadratic. (ii) the P-intercept. [3]

The quadratic model can be found by using two different methods.

(b) Write down three linear equations on a, b, c and hence find the model. [4]

(c) Use your answer in part (a) to find the vertex form $P = a(Q-h)^2 + k$ and then expand to find the quadratic model. [3]

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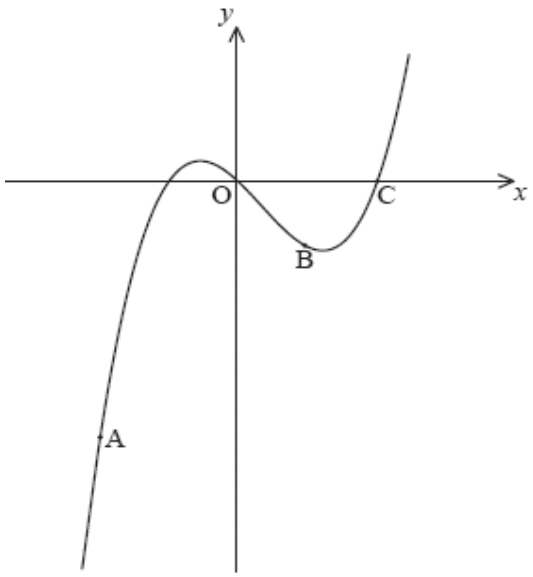
17. [Maximum mark: 9]

Consider the function $f(x) = px^3 + qx^2 + rx$. Part of the graph is shown below.



The graph passes through the origin O and the points A(–2, –8), B(1, –2) and C(2, 0).

(a) Find three linear equations in p, q and r. [3]

(b) Hence find the value of p, of q and of r. [2]

(c) Write down the three roots of the function f. [2]

(d) Solve the inequality $f(x) \geq 0$. [2]

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18. [Maximum mark: 8]

For the point series

x	**1**	**2**	**3**	**4**
y	**3**	**5**	**9**	**13**

two linear models are suggested.

L_1: $y = 3x$ [which passes through (1,3) and (3,9)]

L_2: $y = 3.4x - 1$

(a) Complete the following table of the Squared Residuals and SS_{res} (sum of the squared residuals)

x	y	**Model L_1**		**Model L_2**	
		$y_1 = 3x$	$(y - y_1)^2$	$y_2 = 3.4x - 1$	$(y - y_2)^2$
1	**3**	3	0	2.4	0.36
2	**5**	6	1	5.8	
3	**9**				0.04
4	**13**				
		SUM→		**SUM→**	

[5]

(b) Write a comment about the appropriateness of the two models. [1]

(c) Find the best fit line by using linear regression on your GDC. [2]

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B. Paper 2 questions (LONG)

19. [Maximum mark: 17]

Consider the points A(1,5), B(3, 11), C(6, 20)

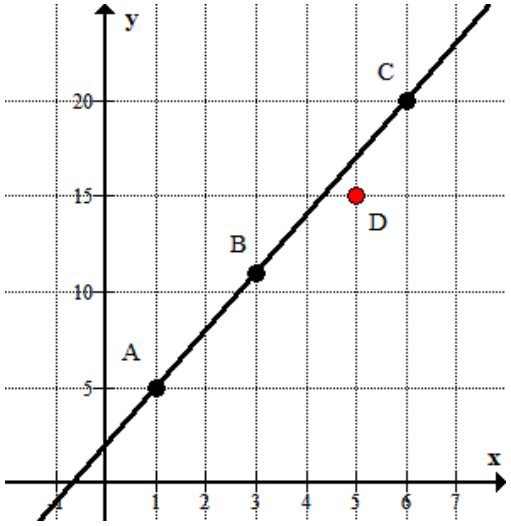
(a) Find

(i) the gradient of the line segment [AB].

(ii) the equation of the line (AB) in the form $y = mx + c$. [4]

(b) Show that the three points A, B and C are collinear. [2]

The following diagram shows the three points A, B, C and the line (AB). An additional point D(5,15) is added.



(c) Show that the point D does not lie on the line (AB). [2]

(d) Explain why the line (AB) is a satisfactory model for the point series A, B, C, D. [2]

(e) Use your GDC to find the best fit line L for the point series A, B, C, D. Give the coefficients in 3s.f. [2]

(f) Complete the following table. Give the squared residuals in 4dp.

x	y	**Estimations**		**Squared Residuals**	
		y_1 by line AB)	y_2 by line L	$(y-y_1)^2$	$(y-y_2)^2$
1	**5**	5		0	
3	**11**	11		0	
5	**15**				
6	**20**				
SS_{res} = SUM OF THE SQUARED RESIDUALS →					

[5]

(g) Write down a comment about the two linear models found. [1]

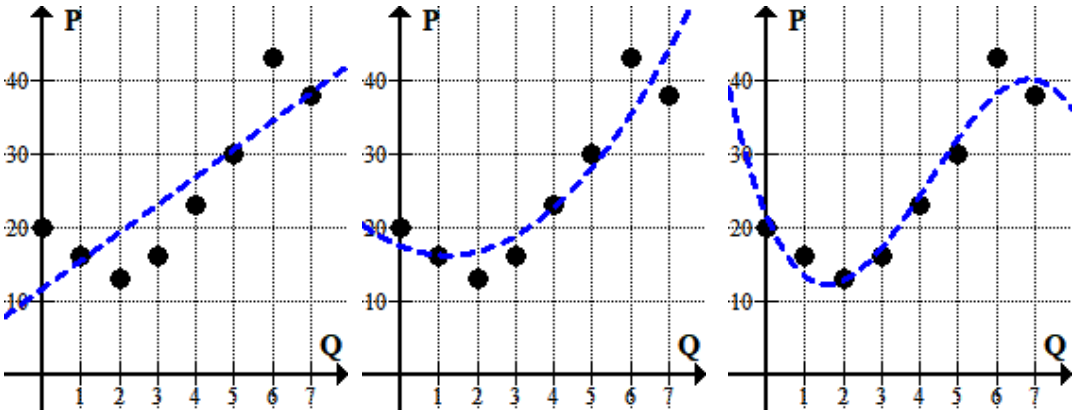
20. [Maximum mark: 10]

Consider the point series

Q	0	1	2	3	4	5	6	7
P	20	16	13	16	23	30	43	38

The following diagrams show three regression models for this point series.

(I) (II) (III)



(a) Using your GDC and regression find the three models:

(i) The linear model (I): $P = aQ + b$

(ii) The quadratic model (II): $P = aQ^2 + bQ + c$

(iii) The cubic model (III): $P = aQ^3 + bQ^2 + cQ + d$.

Give all the coefficients in 3.s.f. [4]

(b) Find the percentage error of each model for the estimation of the P-intercept. [3]

(c) Write down a comment for the appropriateness of the three models. [1]

(d) Suggest a prediction for the value of P when $Q = 8$. [2]

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